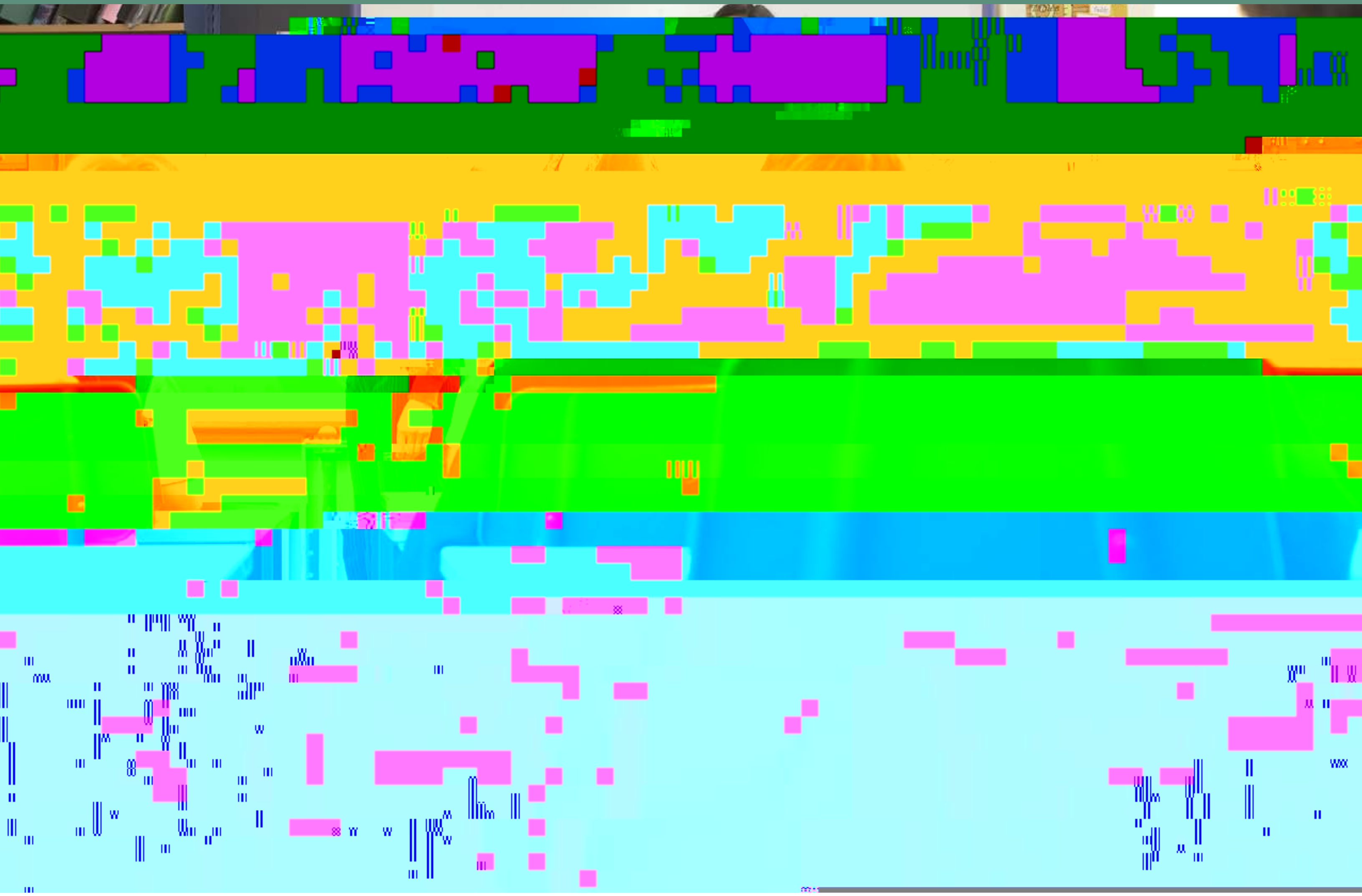
MATHEMATICS INSTRUCTION FOR STUDENTS WITH LEARNING DISABILITIES



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CONTENTS

INTRODUCTION

Historically, mathematics instruction for students with learning disabilities and at-risk learners has not received the same level of consideration and scrutiny from the research community, policy makers, and school administrators as the field of reading. A recent review of the ERIC literature base (Gersten, Clarke, & Mazzocco, 2007) found that the ratio of studies on reading disabilities to mathematics disabilities and difficulties was 5:1 for the years 1996–2005. This was a dramatic improvement over the ratio of 16:1 in the prior decade. Even though this is far from a large body of research, sufficient studies exist to dictate a course of action.

Recently, the Center on Instruction conducted a meta-analysis on the topic of teaching mathematics to students with learning disabilities (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2008). A meta-analysis is a statistical method by which research studies on a particular method of instruction are summarized

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the CtudiyTJT(barticuipats w(r dohe r nityssumh ss aclassroom onrschool s) fnrelrnd oml effective instructional practices in this document are supported by current research findings. Other instructional practices may be effective, but there is, at present, not enough high quality research to recommend their use at this time.

Some of the recommendations listed later in this document (e.g., teach explicitly and use visuals) are age-old teaching practices. While there is nothing new about these practices, research continues to validate them as effective instructional practices for students with learning disabilities and at-risk students, and continued use is warranted. Other instructional methods recommended here, such as using multiple instructional examples and teaching multiple strategies, have also been endorsed in studies that focused on reform-oriented mathematics instruction in general education classes (e.g., Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Rittle-Johnson & Star, 2007). This alignment of teaching methods between special education and general education enables students with learning disabilities to learn meaningfully from general education curricula in inclusive classrooms.

Mathematical Knowledge

Current mathematics researchers emphasize three areas of mathematical abilities (e.g., Kilpatrick, Swafford, & Findell, 2001; Rittle-Johnson & Star, 2007; Bottge, Rueda, LaRoque, Serlin, & Kwon, 2007). They are:

- procedural knowledge,
- procedural flexibility, and
- conceptual knowledge.

Procedural knowledge refers to knowledge of basic skills or the sequence of steps needed to solve a math problem. Procedural knowledge enables a student to execute the necessary action sequences to solve problems (Rittle-Johnson & Star, 2007).

Procedural flexibility refers to knowing the many different ways in which a particular problem can be solved. Students with a good sense of procedural flexibility know that a given problem can be solved in more than one way, and can solve an unknown problem by figuring out a possible solution for that problem.

Conceptual knowledge is a grasp of the mathematical concepts and ideas that are not problem-specific and therefore can be applied to any problem-solving situation. Conceptual understanding is the over-arching understanding of mathematical concepts and ideas that one often refers to as a "good mathematical sense."

It is reasonable to extrapolate from this small but important body of research that such an emphasis would also benefit students with disabilities and at-risk students. Recent studies have attempted to address reform-oriented math instruction in special education settings. Researchers such as Woodward (e.g., Woodward, Monroe, & Baxter, 2001) and Van Luit (Van Luit & Naglieri, 1999) have endeavored to address the issue of procedural flexibility in their

Our document guides K–12 teachers of students with disabilities and at-risk students in their selection and use of effective mathematics instructional methods. For each of the seven recommendations, we explain what works, describe how the practice should be done, and summarize the evidence supporting the recommendation.

Recommendation 1:
Teach students using *explicit instruction* on a regular basis.

Explicit instruction, a mainstay feature in many special education programs, includes teaching components such as:

- clear modeling of the solution specific to the problem,
- thinking the specific steps aloud during modeling,
- presenting multiple examples of the problem and applying the solution to the problems, and
- providing immediate corrective feedback to the students on their accuracy.

When teaching a new procedure or concept, teachers should begin by modeling and/or thinking aloud and working through several examples. The teacher emphasizes student problem solving using the modeled method, or by using a model that is consonant with solid mathematical reasoning. While modeling the steps in the problem (on a board or overhead), the teacher should verbalize the procedures, note the symbols used and what they mean, and explain any decision

Recommendation 2:
Teach students using *multiple instructional examples*.

Example selection in teaching new math skills and concepts is a seminal idea that is strongly emphasized in the effective instruction literature (e.g., Ma, 1999; Rittle-Johnson & Star, 2007; Silbert, Carnine, & Stein, 1989). Teachers need to spend some time planning their mathematics instruction, particularly focusing on selecting and sequencing their instructional examples. The goal is to select a range of multiple examples of a problem type. The underlying intent is to expose students to many of the possible variations and at the same time highlight the common but critical features of seemingly disparate problems. For example, while teaching students to divide a given unit into half, a variety of problems can be presented that differ in the way the critical task of half is addressed in the problems (i.e., use the symbol for half; use the word half, use the word one-half; etc.) (Owen & Fuchs, 2002).

Multiple examples can be presented in a specified sequence or pattern such as concrete to abstract, easy to hard, and simple to complex. For example, fractions and algebraic equations can be taught first with concrete examples, then with pictorial representations, and finally in an abstract manner (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Witzel, Mercer, Miller, 2003). Multiple examples can also be presented by systematically varying the range presented (e.g., initially teaching only proper fractions vs. initially teaching both proper and improper fractions).

Sequencing of examples may be most important during early acquisition of new skills when scaffolding is needed for student mastery and success. The range of examples taught is probably most critical to support transfer of learned skills to new situations and problems. In other words, if the teacher teaches a wide range of examples, it will result in the learner being able to apply a skill to a wider range of problem types. Both of these planning devices (sequence and range) should be considered carefully when teaching students with LD.

Summary of Evidence to Support Recommendation 2

Meta-analysis of Mathematics Intervention Research for Students with LD

COI examined 9 studies on range and sequence of examples (all RCTs). The mean effect size of 0.82 was statistically significant ($p<.001$; 95% CI = 0.42 to 1.21).

National Mathematics Advisory Panel

The panel reviewed 26 high quality studies (mostly RCTs) on effective instructional approaches for students with learning disabilities and low-achieving students. The panel recommends that teachers, as part of explicit instruction, carefully sequence problems to highlight the critical features of the problem type (National Mathematics Advisory Panel, 2008).

Recommendation 3:
Have students *verbalize decisions and solutions* to a math problem.

Encouraging students to verbalize, or think-aloud, their decisions and solutions to a math problem is an essential aspect of scaffolded instruction (Palincsar, 1986). Student verbalizations can be problem-specific or generic. Students can verbalize the specific steps that lead to the solution of the problem (e.g., I need to divide by two to get half) or they can verbalize generic heuristic steps that are common to problems (e.g., Now I need to check my answer). Students can verbalize the steps in a solution format (First add the numbers in the *units* column. Write down the answer. Then add numbers in the *tens* column…) (Tournaki, 2003) or in a self-questioning/answer format (What should I do first? I should…) (Pavchinski, 1998). Students can verbalize during initial learning or as they are solving, or have solved, the problem.

Many students with learning disabilities are impulsive behaviorally and when faced with multi-step problems frequently attempt to solve the problems by randomly combining numbers rather than implementing a solution strategy step-by-step. Verbalization may help to anchor skills and strategies both behaviorally and mathematically. Verbalizing steps in problem solving may address students' impulsivity directly, thus suggesting that verbalization may facilitate students' self-regulation during problem solving.

Summary of Evidence to Support Recommendation 3

Meta-analysis of Mathematics Intervention Research for Students with LD

COI examined 8 studies in the area of student verbalizations (7 RCTs and 1 QED). The mean effect size of 1.04 was statistically significant ($p<.001$; 95% CI = 0.42 to 1.66).

National Mathematics Advisory Panel

The panel reviewed 26 high quality studies (mostly RCTs) on effective instructional approaches for students with learning disabilities and low-achieving students. The panel recommends that teachers, as part of explicit instruction, allow students to think aloud about the decisions they make while solving problems (National Mathematics Advisory Panel, 2008).

Recommendation 4:
Teach students to *visually represent the information* in the math problem.

Visual representations (drawings, graphic representations) have been used intuitively by teachers to explain and clarify problems and by students to understand and simplify problems. When used systematically, visuals have positive benefits on students' mathematic performance.

Recommendation 5:
Teach students to solve problems using *multiple/heuristic strategies*.

Instruction in multiple/heuristic strategies is part of a contemporary trend in mathematics education (e.g., Star & Rittle-Johnson, in press). Using heuristics shows some promise with students with learning disabilities. Multiple/heuristic strategy instruction has been used in addressing computational skills, problem solving, and fractions.

A heuristic is a method or strategy that exemplifies a generic approach for solving a problem. For example, a heuristic strategy can include steps such as "Read the problem. Highlight the key words. Solve the problems. Check your work." Instruction in heuristics, unlike direct instruction, is not problem-specific. Heuristics can be used in organizing information and solving a range of math problems. They usually include student discourse and reflection on evaluating the alternate solutions and finally selecting a solution for solving the problem. For example, in the Van Luit and Naglieri (1991) study, the teacher first modeled several strategies for solving a computational problem. However, for most of the lesson, the teacher's task was to lead the discussion in the direction of using strategies and to facilitate the discussion of the solutions provided by the students. Each student was free to select a strategy for use, but the teacher assisted the children in discussion and reflection about the choices made.

Similarly, in the Woodward (2006) study, students were taught multiple fact strategies. Daily lessons consisted of introduction of new strategies or review of old strategies. Students were not required to memorize the strategies. They were, however, encouraged to discuss the strategy and contrast it with previously taught strategies. For example, students were shown that since 9 X 5 has the same value as 5 X 9, they were free to treat the problem as either nine fives or five nines. They also were shown that this was equivalent to 10 fives minus one five, and that this could be a faster way to do this problem mentally. Thus a variety of options were discussed with the students.

Summary of Evidence to Support Recommendation 5

Meta-analysis of Mathematics Intervention Research for Students with LD

COI examined 4 studies in the area of multiple/heuristic strategy instruction (3 RCTs and 1 QED). The mean effect size of 1.56 was statistically significant ($p<.001$; 95% CI = 0.65 to 2.47).

Recommendation 6:
Provide *ongoing formative assessment data and feedback* to teachers.

Ongoing formative assessment and evaluation of students' progress in mathematics can help teachers measure the pulse and rhythm of their students' growth and also help them fine-tune their instruction to meet students' needs. Teachers can administer assessments to their group of students and then a computer can provide them with data depicting students' current mathematics abilities.

Providing teachers with information regarding their students' progress in mathematics has beneficial effects on the mathematics performance of those same students. However, greater benefits on student performance will be observed if teachers are provided with not only performance feedback information but also instructional tips and suggestions that can help teachers decide what to teach, when to introduce the next skill, and how to group/pair students. For example, teachers can be given a set of written questions to help them use the formative assessment data for adapting and individualizing instruction. These written questions could include "On what skill(s) has the student improved compared to the previous two-week period?" or "How will I attempt to improve student performance on the targeted skill(s)?"

Teachers can respond to these questions and address them again when new assessment data becomes available (Allinder, Bolling, Oats, & Gagnon, 2000). Teachers can also be provided

LIST OF RECOMMENDATIONS

1: Teach students using *explicit instruction* on a regular basis.

2: Teach students using *multiple instructional examples.*

3: Have students *verbalize decisions and solutions* to a math problem.

4: Teach students to *visually represent the information* in the math problem.

5: Teach students to solve problems using *multiple/heuristic strategies.*

6: Provide *ongoing formative assessment data and feedback* to teachers.

7: Provide *peer-assisted instruction* to students.

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